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Array Shape Tracking Using Active Sonar Reverberation

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Array Shape Tracking from Active Sonar Clutter

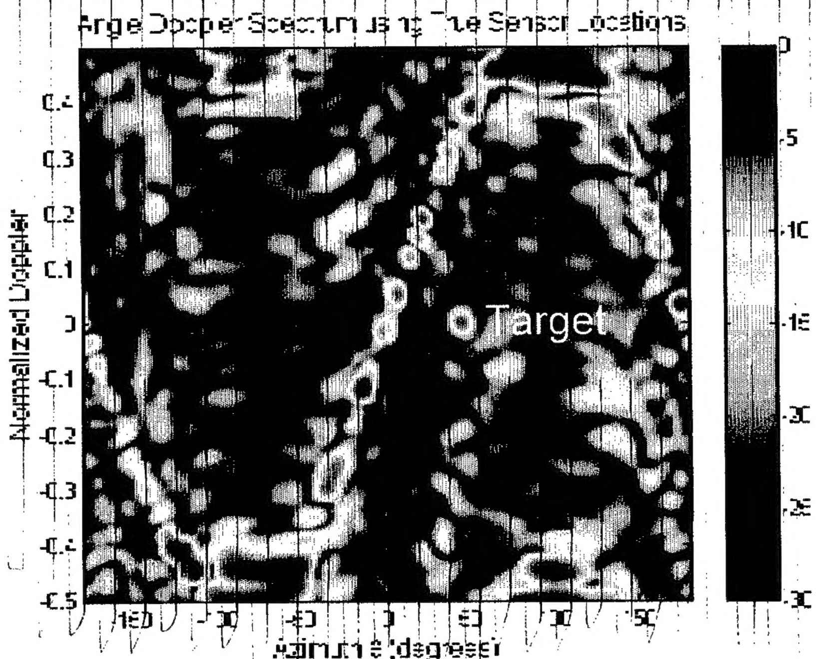
OBJECTIVE: To estimate and track the shape of a towed distorted-linear array using reverberation from active sonar pings as a distributed source of opportunity.

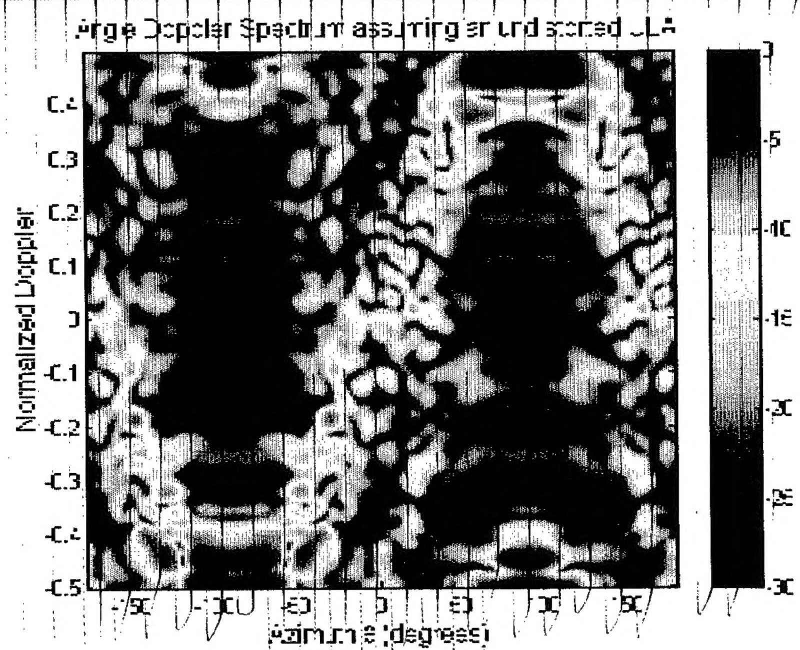
BACKGROUND:

- Array shape estimation using heading and depth sensors has been previously developed, e.g. Gray *et al.*(1993), by applying Kalman filtering with a state equation derived from spatio-temporal discretization of the simplified Paidoussis equation (water-pulley model) for array motion.
- Acoustic sources of opportunity have been employed for array shape estimation by fitting transverse element displacements to measuring inter-element phase differences e.g. Owsley (1980).
- For mid-frequency active sonar arrays the above methods may be precluded since it may be infeasible to instrument the array with a sufficient number of heading sensors and strong point sources of opportunity may not always be available in the presence of strong reverberation.
- Clutter has previously been used for element gain and phase calibration of a uniform linear airborne radar array (Robey, Fuhrmann, and Krisch, 1994) but not for array shape estimation.
- We propose constrained maximum likelihood array shape estimation from clutter (ASEC) which uses an array shape-dependent model of spatially-distributed, Doppler-spread reverberation.
- Array shape parameters are tracked within and across pings by using ML ASEC heading estimates as input to a Kalman filter whose state equation incorporates a dynamical model for array motion.

Motivation for Array Shape Estimation

- Detection of slowly-moving targets with Doppler-sensitive waveforms is limited by beamformer sidelobes.
- Uncompensated array distortion causes increased clutter rank and target masking.
- Simulation example angle-Doppler spectrum from *perfectly compensated* array (left) vs. uncompensated *distorted* array (right). Note target masked by reverberation due to beamformer sidelobes of uncompensated array

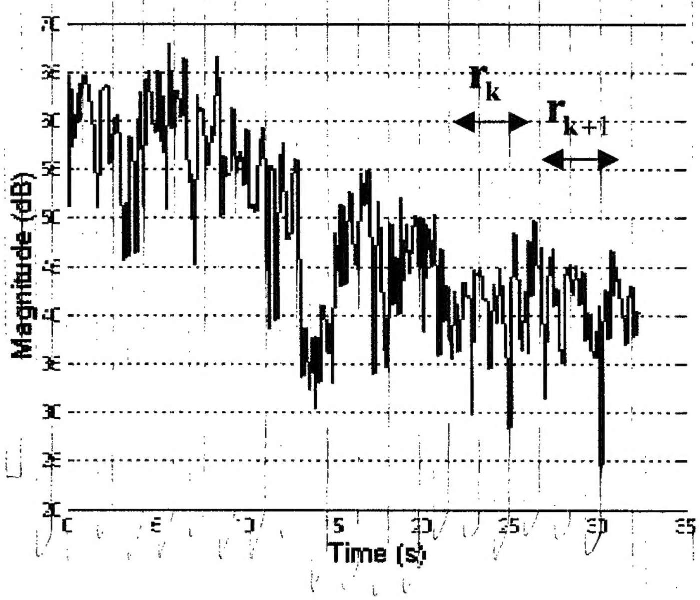


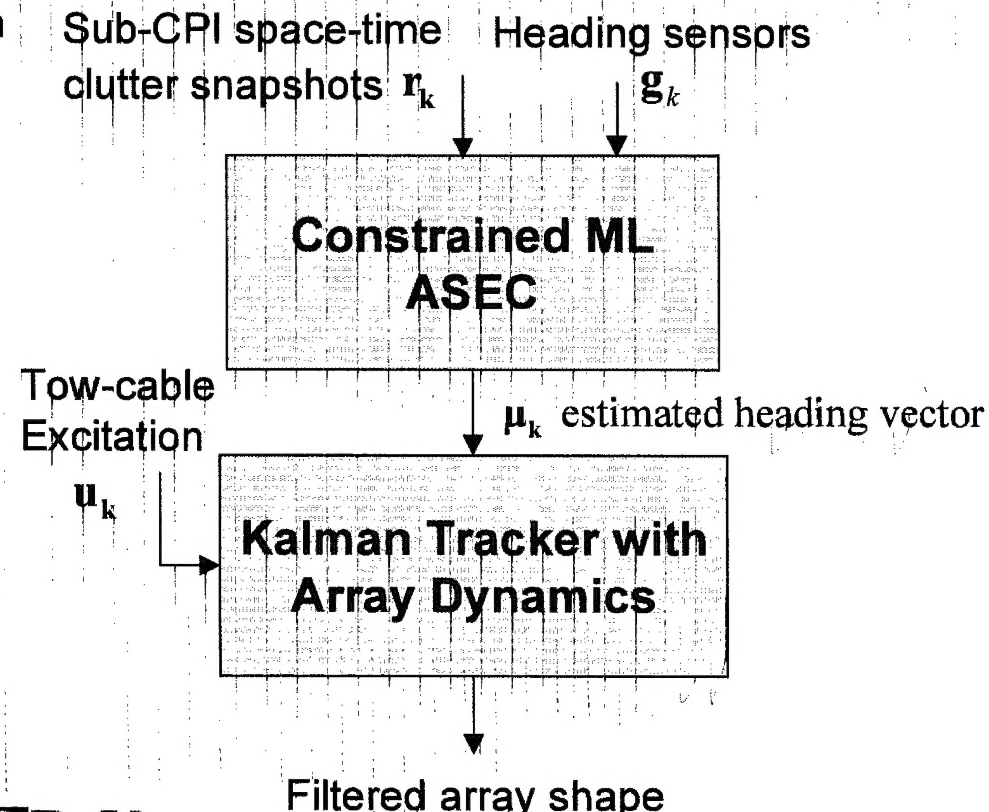


Array Shape Estimation from Clutter Concept

- Constrained ML ASEC assimilates clutter data with a few heading sensor outputs to obtain array shape parameters from each sub-pulse length space-time snapshot.
- Kalman tracker filters sub-CPI shape coefficients using a model for array dynamics.

Example SWAC-3 Reverberation Return





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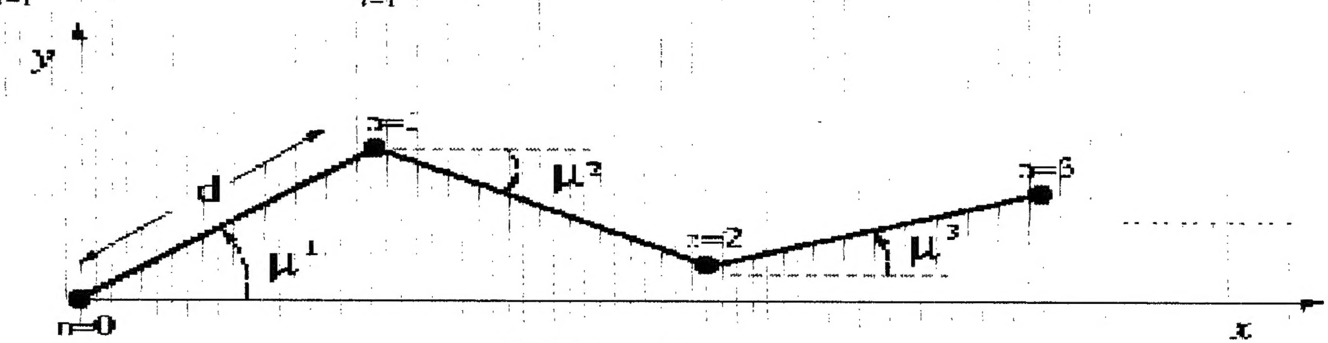
Reverberation Received at a Distorted Array

The clutter data from the n^{th} sensor located at (x_n, y_n) at time $t_m = \tau + mT_r$ for a distorted array moving with velocity v_a along the x direction is modeled as:

$$r_{nm} = \sum_{\theta_k,\phi_l} \alpha(\theta_k,\phi_l) e^{j\frac{2\pi}{\lambda}(\sin\theta_k \cos\phi_l \cdot x_n + \cos\theta_k \cos\phi_l \cdot y_n) + j2\pi\frac{2v_a}{\lambda}\sin\theta_k \cos\phi_l \cdot mT_r}$$

where $\alpha(\theta_k,\phi_l)$ is the complex Gaussian scatter amplitude from clutter at azimuth, θ_k, and multipath elevation, ϕ_l.

Sensor coordinates can be expressed in terms of heading μ and inter-element spacing d as $x_n = d\sum_{i=1}^{n} \cos(\mu_i)$, $y_n = d\sum_{i=1}^{n} \sin(\mu_i)$, $0 \le n \le N-1$, $(x_0, y_0) \equiv (0,0)$.



Space-Time Reverberation Model

The space-time data snapshot at time t_k consisting of clutter from all azimuths $\theta_i \in [-\pi,\pi)$, $1 \le i \le N_\theta$ and elevation angles $|\phi_j| \le \phi_{\max}$, $1 \le j \le N_\phi$, can be written as

$$\mathbf{r}_k = \mathbf{V}(\boldsymbol{\mu}_k)\boldsymbol{\eta}_k + \boldsymbol{\varepsilon}_k$$

where $\boldsymbol{\mu}_k = \left[\mu_k^{\,1} \;\cdots\; \mu_k^{\,N-1}\right]^T$, $\mathbf{V}(\boldsymbol{\mu}_k) = \left[\mathbf{v}(\theta_1,\phi_1,\boldsymbol{\mu}_k) \;\cdots\; \mathbf{v}(\theta_{N_\theta},\phi_{N_\phi},\boldsymbol{\mu}_k)\right]$ is the clutter steering matrix, $\boldsymbol{\eta}_k$ represents unknown scattering, and noise $\boldsymbol{\varepsilon}_k \in {}^{MN\times 1}$ has covariance $\sigma_\varepsilon^{\,2} I_{MN}$.

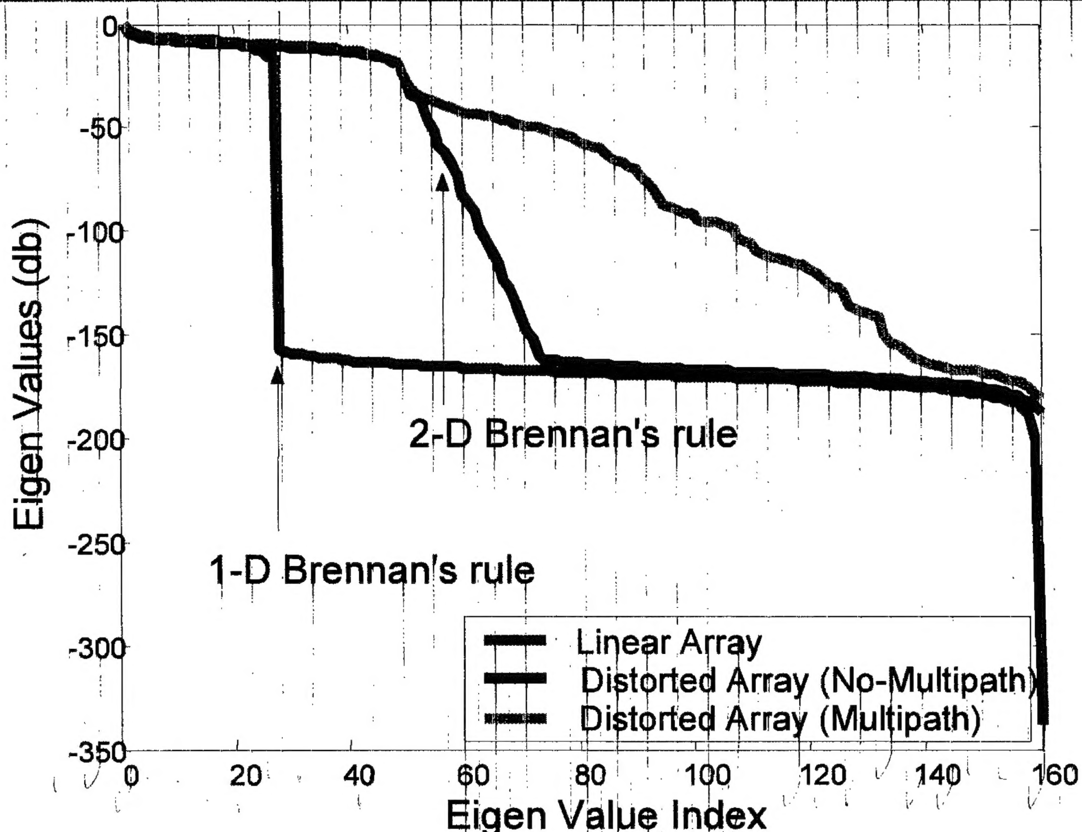
The $\left((i-1)N_\theta + j\right)^{th}$ column of $\mathbf{V}(\boldsymbol{\mu}_k)$ is $\mathbf{v}(\theta_i,\phi_j,\boldsymbol{\mu}_k) = \mathbf{b}(\varpi_{ij}) \otimes \mathbf{a}(\theta_i,\phi_j,\boldsymbol{\mu}_k)$ which represents the return from a single clutter patch at location (θ_i,ϕ_j).

Array shape parameters $\boldsymbol{\mu}_k$ are assumed constant over the sub-CPI.

Array shape parameters $\boldsymbol{\mu}_k$ estimated by fitting the low rank $(< MN)$ clutter subspace of $\mathbf{V}(\boldsymbol{\mu}_k)$ to the observed received space-time snapshot, $\mathbf{r}_k$.

Clutter Eigenvalues for a Distorted Linear Array

- Eigenvalues of the asymptotic space-time covariance matrix for a modestly distorted array versus ideal uniform linear array for N=32 and M=5. Observe that rank inflation with 0 dB backlobes agrees with 2-D Brennan's rule [Varadarajan and Krolik, 2002].



Maximum Likelihood Array Shape Estimation

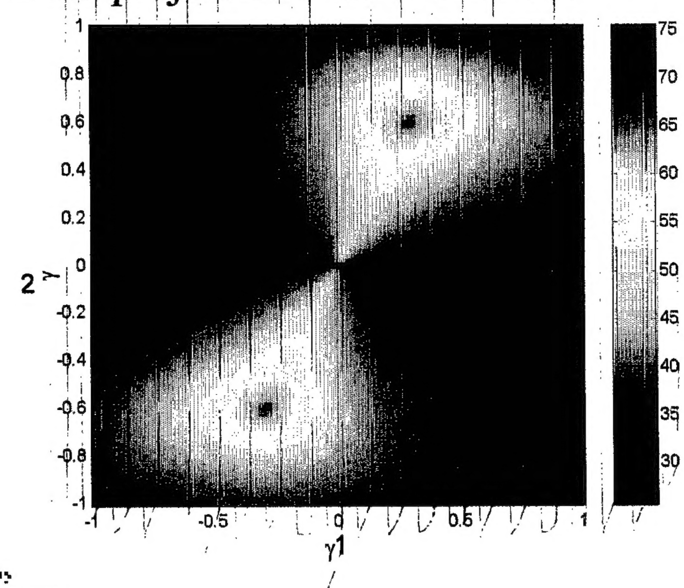
The N - 1 array shape headings can be parameterized by a low dimensional subspace using $(N-1)\times(L \quad N)$ arbitrary but known shape basis as $\boldsymbol{\mu}_k = \boldsymbol{\Psi}\boldsymbol{\gamma}_k$.

The likelihood function for $\boldsymbol{\gamma}_k$ for the model reduces to minimizing the projection:

$$\hat{\boldsymbol{\gamma}}_k = \arg\min_{\gamma_k} f(\boldsymbol{\gamma}_k) = \left\|\mathbf{P}(\boldsymbol{\gamma}_k)\mathbf{r}_k\right\|^2$$

where $\mathbf{P}(\boldsymbol{\gamma}_k) = \mathbf{I} - \mathbf{V}(\boldsymbol{\gamma}_k)\left(\mathbf{V}(\boldsymbol{\gamma}_k)^H\mathbf{V}(\boldsymbol{\gamma}_k)\right)^{-1}\mathbf{V}^H(\boldsymbol{\gamma}_k)$ is the projection matrix onto the orthogonal complement of the clutter subspace.

Evaluation of $\log(f(\boldsymbol{\gamma}))$ (right) for $L = 2$ demonstrates left-right ambiguity in shape estimate corresponding to mirrored solutions about the array axis.



Constrained ML ASEC Algorithm

The left-right shape ambiguity can be resolved with knowledge of the position or heading of a single off-axis sensor which can be expressed as a linear constraint on the shape basis coefficients as

$$\mathbf{c}^{T}\boldsymbol{\gamma}_k = g.$$

The ML ASEC estimate can be obtained iteratively using a gradient projection approach (e.g. Frost (1972)):

$$\hat{\boldsymbol{\gamma}}_k^{\,j+1} = \boldsymbol{\gamma}_c + \mathbf{P}_c^{\perp}\left(\hat{\boldsymbol{\gamma}}_k^{\,j} - \xi \mathbf{f}'^{\,j}\right)$$

where $\boldsymbol{\gamma}_c = \mathbf{P}_c \hat{\boldsymbol{\gamma}}_k^{\,j} = \mathbf{c}(\mathbf{c}^T\mathbf{c})^{-1} g$ is the projection of the current solution (j[th] iteration) onto the constraint subspace, $\mathbf{P}_c^{\perp} = \mathbf{I} - \mathbf{P}_c$, $\left[\mathbf{f}'^{\,j}\right]_i = \left[\dfrac{\partial f(\boldsymbol{\gamma})}{\partial[\boldsymbol{\gamma}_k]_i}\right]_{\boldsymbol{\gamma}=\hat{\boldsymbol{\gamma}}^j} = -2\Re\left\{\mathbf{r}_k^{\,H}\mathbf{P}(\boldsymbol{\gamma}_k)\mathbf{V}_i(\boldsymbol{\gamma}_k)\left(\mathbf{V}(\boldsymbol{\gamma}_k)^H\mathbf{V}(\boldsymbol{\gamma}_k)\right)^{-1}\mathbf{V}(\boldsymbol{\gamma}_k)^H\mathbf{r}_k\right\}$, $0<\xi<1$.

The matrix $\mathbf{V}_i(\boldsymbol{\gamma}_k) = \dfrac{\partial \mathbf{V}(\boldsymbol{\gamma}_k)}{\partial[\boldsymbol{\gamma}_k]_i}$ can be computed analytically by assuming the temporal component of the steering matrix is independent of array shape distortion over a sub-CPI, and an analytic form for $\mathbf{V}(\boldsymbol{\gamma}_k)$ obtained by judicious sampling of the clutter wavenumber spectrum.

The rank of $\mathbf{V}(\boldsymbol{\gamma}_k)$ is assumed constant and can be chosen so that it does not change over the set of possible array distortions.

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Incorporating Array Dynamics into ASEC

- A Kalman filter can be used to track the array shape coefficients across multiple sub-CPI intervals during a ping.

- The state vector $\boldsymbol{\mu}_{k+1}$ can be related to $\boldsymbol{\mu}_k$ using the water-pulley model:

$$\boldsymbol{\mu}_{k+1} = \mathbf{F}\boldsymbol{\mu}_k + \mathbf{u}_k + \mathbf{v}_{1k}$$

where $\mathbf{F} = (1-\rho)\mathbf{I} + \rho\mathbf{L}$ is the state transition matrix, $\mathbf{u}_k = \begin{bmatrix} \mu_k & \mathbf{0}_{1\times N-2} \end{bmatrix}^H$ is the tow-cable driving term, $\mathbf{v}_{1k}$ and represents white state noise with $\sigma_{v_1}{}^2\mathbf{I}$. It can be shown that the displacement velocity along the array is determined by $\rho = \tilde{\rho}\beta/2$.

- An observation equation can be defined using the MLE $\mathbf{z}_k = \hat{\boldsymbol{\mu}}_k$ from each sub-CPI:

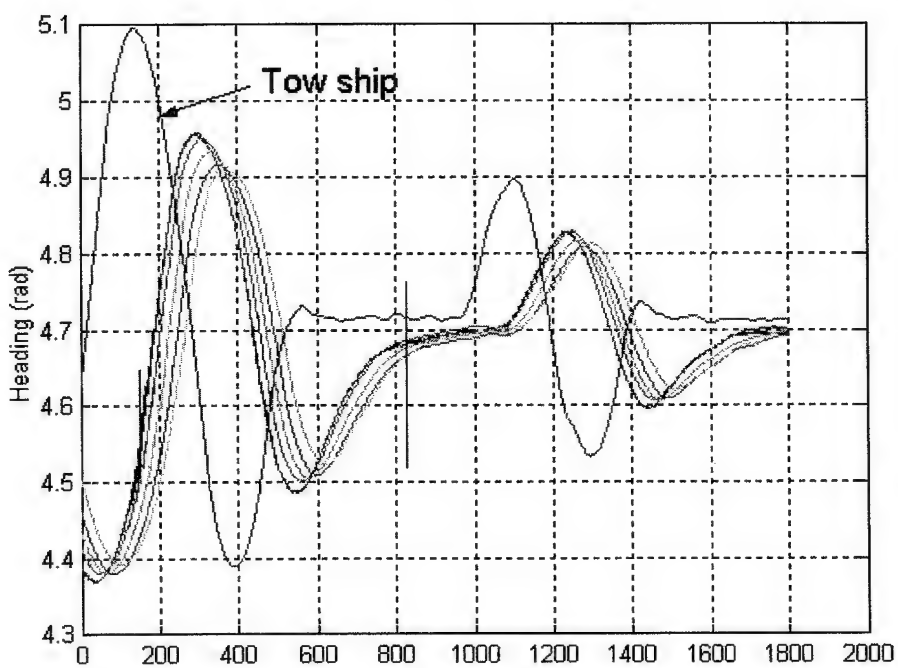
$$\mathbf{z}_k = \boldsymbol{\mu}_k + \mathbf{v}_{2k}$$

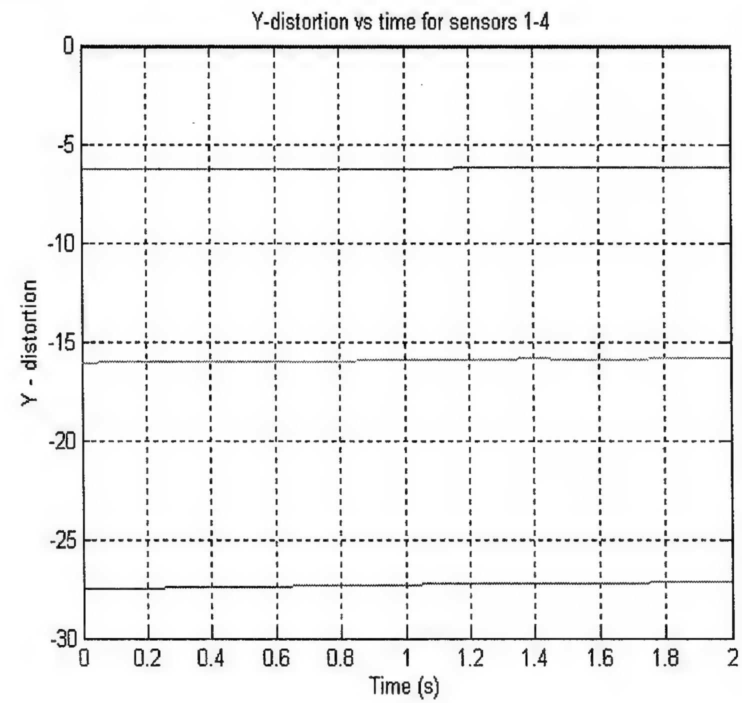
where $\mathbf{v}_{2k}$ is the measurement noise with covariance $\sigma_{v_2}{}^2\mathbf{I}$.

- The predicted MMSE array shape is used to initialize the next ASEC MLE search.

Array Dynamical Model Validation Using TB-29 Data

- Real heading sensor data from a TB-29 array (left) vs. time (sec), courtesy of Bruce Newhall at APL/JHU, indicates water pulley model valid for mild maneuvers.
- Transverse distortion over CPI (right) validates assumption of rigid short-time motion.





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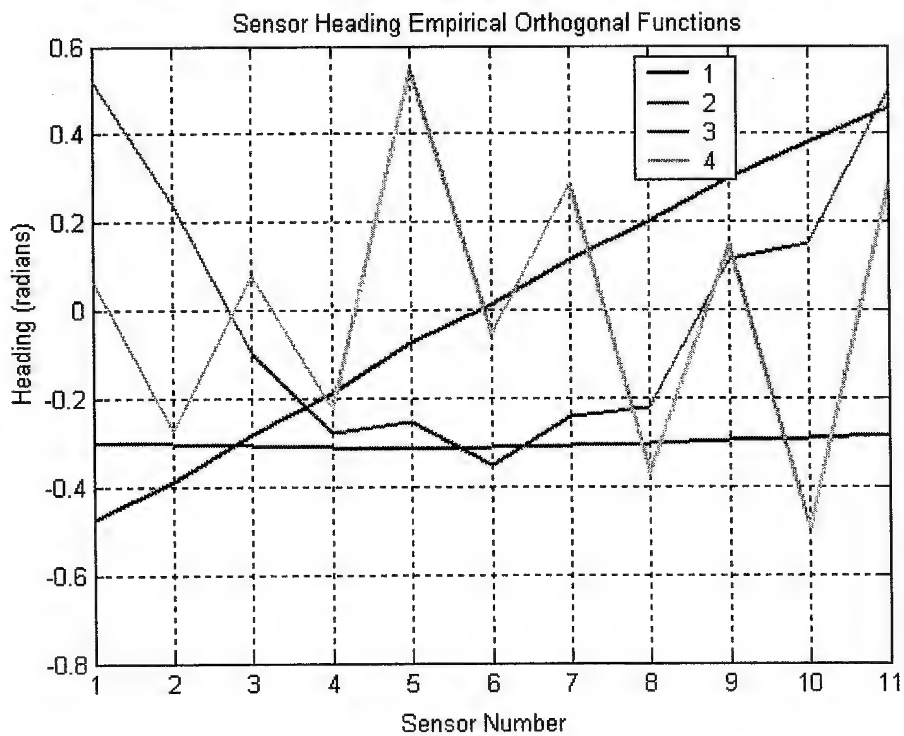
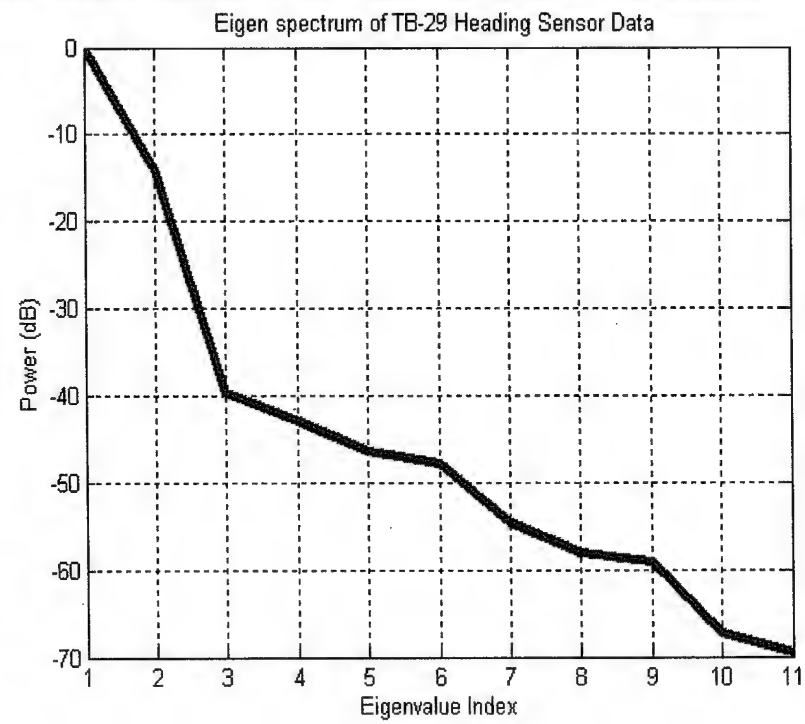
TB-29 Heading Sensor Eigenbasis

- Empirical orthogonal basis vectors (left) for headings derived from 6 heading sensor TB-29 data (interpolated to 11 sensors) demonstrate characteristic behavior.
- TB-29 heading sensor eigen-spectrum (right) indicates most of the variation captured by less than 4 modes.





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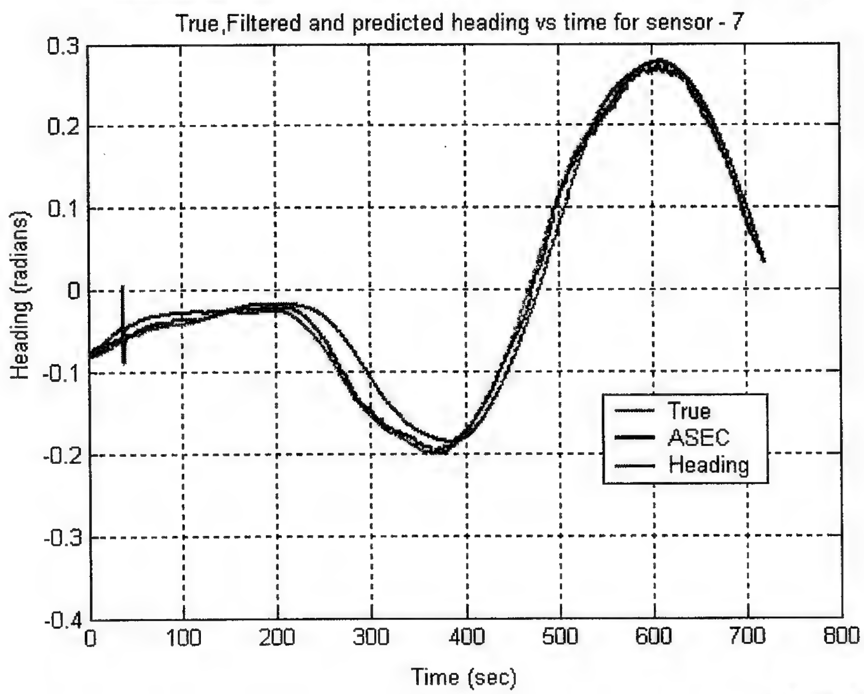
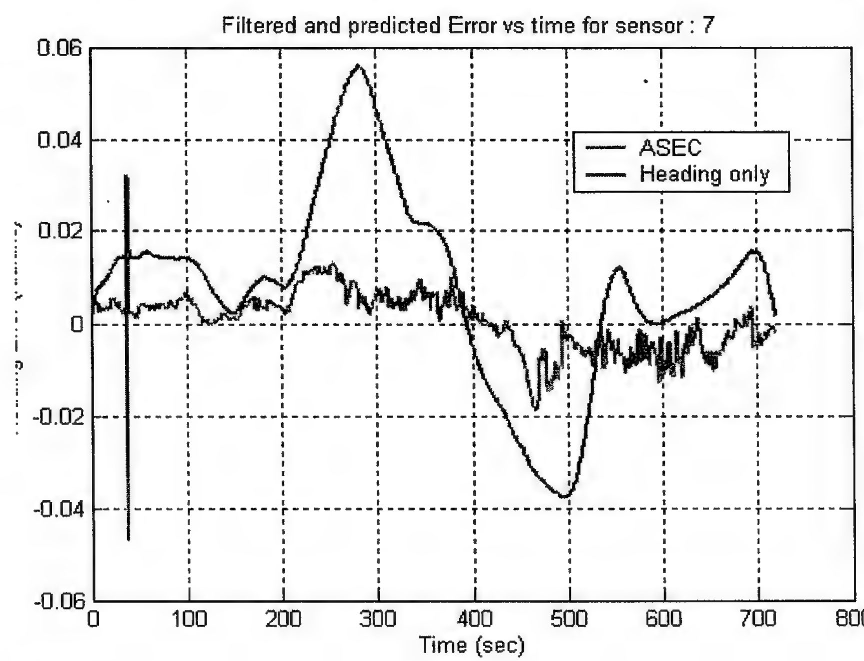
ASEC versus Heading Only Shape Estimation

- Illustrative simulation comparison of ASEC versus headings only tracking of a mild maneuver as seen in the middle of the array based on TB-29 heading data (left).
- Simulation assumes ASEC with 1.2 s. moving window sub-CPI, 4 EOF basis, space-time snapshot dimension 144 and clutter rank of 55.
- Heading (left) and error (right) for ASEC and 2 heading-sensor tracking as in MFTA.





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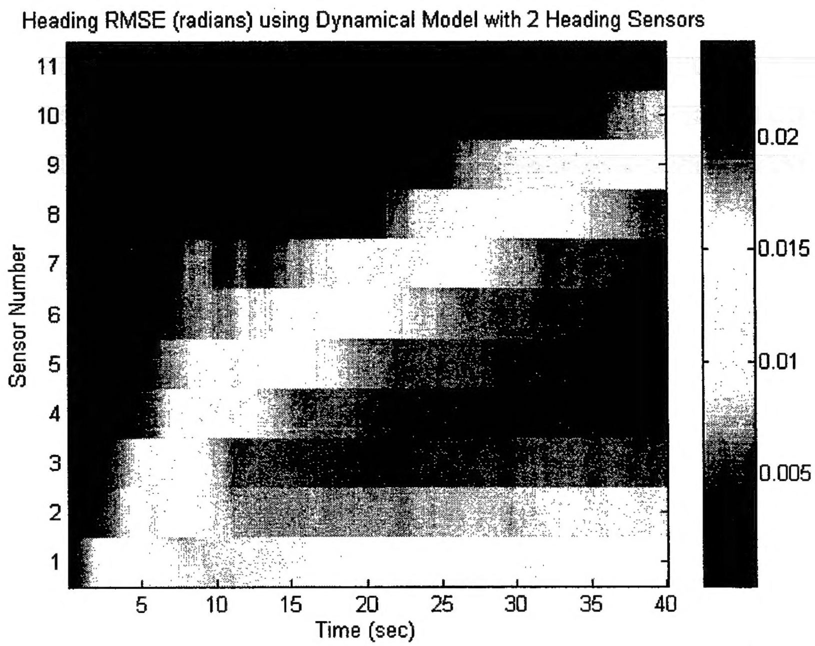
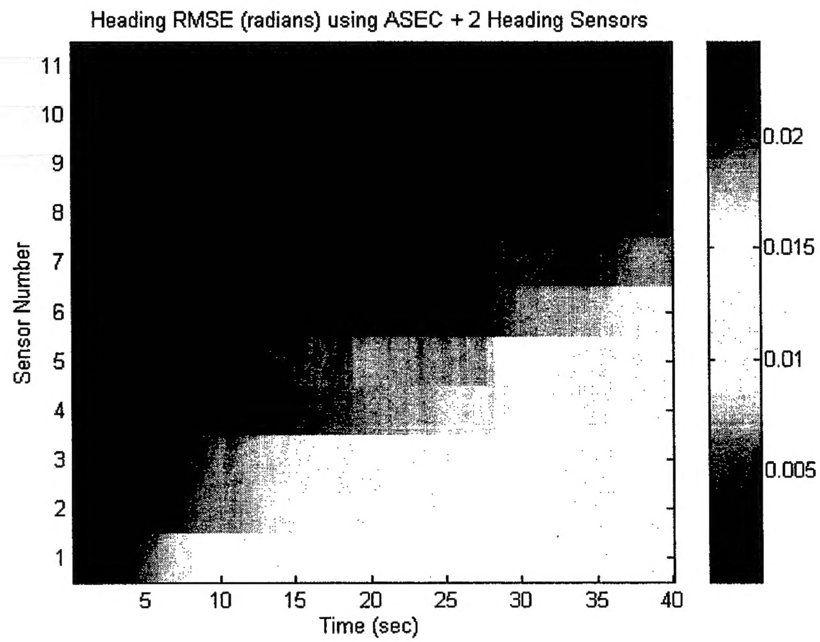
RMS Heading Error along the Array verus Time

- Simulated RMSE for 2-sensor headings-only tracking error (left) and ASEC (right) over 40 second ping along the array, ensemble averaged over initial condition perturbations and clutter realizations.
- Observe propagation of error down the array with time using water-pulley model (left). Error largest in the middle of the array since headings known at the end and at tow-point offset from first sensor.





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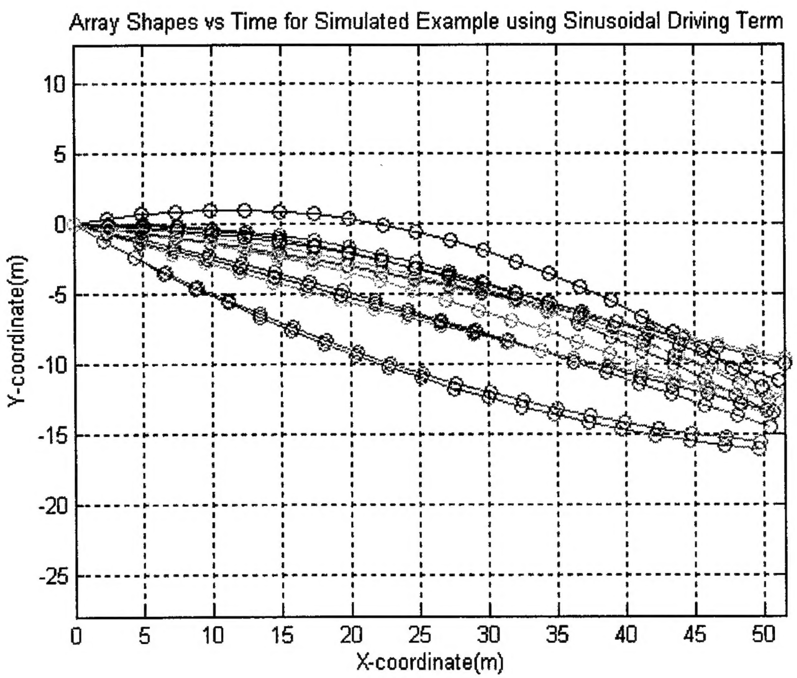
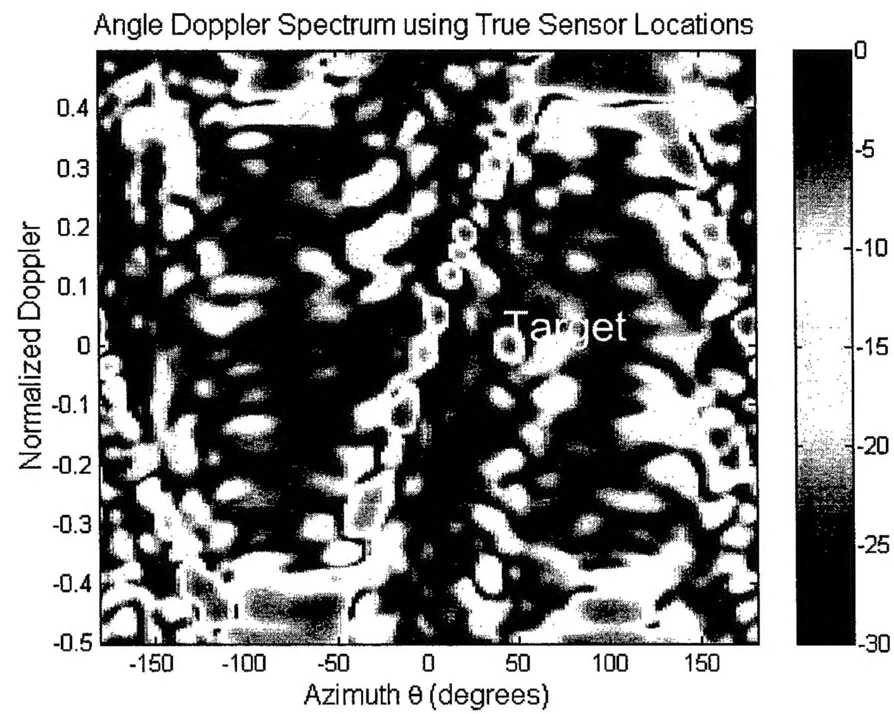
Angle-Doppler Spectrum for Distorted Array

- Impact of ASEC tracking on sonar performance illustrated by conventional beamforming-Doppler spectra outputs simulated for continuous array motion.
- Time-evolving simulated array shapes based on scaled TB-29 motion model (left). Angle-Doppler spectrum with 10 dB target for perfect array compensation (right).





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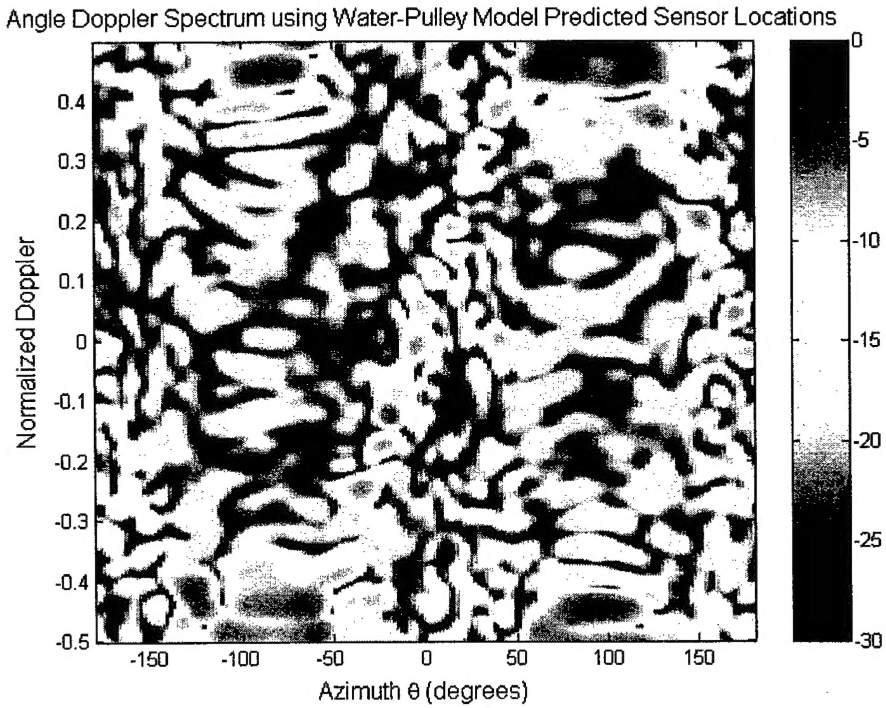
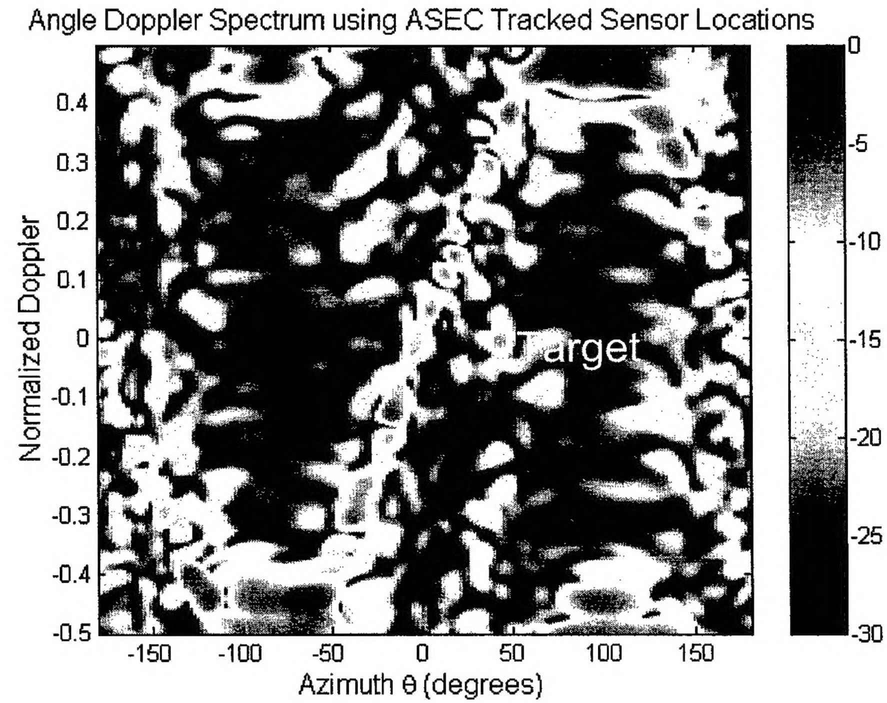
ASEC vs. Headings-only Tracking Angle-Doppler Spectra

- Spectrum for conventional array tracking for WP model with 2 heading sensors (left).
- Spectrum for ASEC tracking with 4 basis functions, 1.2 s. sub-CPI sliding window, and CNR = 30 dB (right). Target visible at zero-Doppler and 45 degree bearing.





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ASEC Summary and Future Work

- Distortion of nominally linear arrays will often result in a substantial increase in spatial sidelobe levels which can mask slowly-moving targets.
- ML estimation of array shape is facilitated by fitting a reduced-rank reverberation model to a sliding window of sub-CPI space-time snapshots over the extent of each sonar return.
- Shape ambiguities can be resolved by efficiently maximizing likelihood subject to a constraint which incorporates measurements from at least a single heading sensor.
- Constrained ML ASEC heading estimates are used as inputs to a Kalman tracker incorporating an array dynamical model and driven by a tow-cable heading sensor output.
- Simulations using array motion scaled from real TB-29 heading data suggests that ASEC tracking can facilitate improved array compensation compared to headings-only tracking.
- Future work will include ASEC performance evaluation with real 53C/MFTA data.